# SS08: Fuzzy Implication Functions

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In recent years, fuzzy implication functions have become one of the main research lines of the fuzzy logic community. These logical connectives are the generalization of the classical two-valued implication to the infinite-valued setting. In addition of modelling fuzzy conditionals, they are also used to perform backward and forward inferences in different fuzzy rules based systems. Moreover, they have proved to be useful not only in fuzzy control and approximate reasoning, but also in many other fields like Multi-Valued Logic, Image Processing, Data Mining, Computing with Words and Rough Sets, among others.

Due to this great quantity of applications, fuzzy implication functions have attracted the efforts of many researchers also from the theoretical perspective focusing on problems whose solutions provide important insights from the point of view of their applications. Therefore, this special session seeks to bring together researchers interested in recent advances in the theory of fuzzy implication functions, concerning, among others, characterizations, representations, generalizations and their relationships with fuzzy negations, triangular norms, uninorms and other fuzzy logic connectives. The joint 17th World Congress of International Fuzzy Systems Association and 9th International Conference on Soft Computing and Intelligent Systems (IFSA-SCIS 2017), as a conference which focuses on bringing together scientists working on Fuzzy Sets and Soft Computing methods, constitutes an excellent opportunity to progress particularly on the knowledge of fuzzy implication functions.

The three accepted papers in this special session deal with several aspects of fuzzy implication functions.

The first paper, entitled "On Fuzzy Implication Functions Defined Using Powers of Continuous t-norms" and written by Sebastia Massanet, Jordi Recasens and Joan Torrens, focuses on the recently introduced $T$-power based implications and presents some characterizations of this family of fuzzy implication functions up to compositions with increasing mappings. $T$-power based implications satisfy, among other important properties, the invariance of the truth value of the fuzzy conditional when both the antecedent and the consequent are modified using the same linguistic quantifier modeled through powers of continuous t-norms. This property, which is closely related to approximate reasoning, is proved to be such a strong property which allows to characterize, with some other minimal properties, this family of fuzzy implication functions.

In the following paper "On some Constructions of Ordinal Sums of Fuzzy Implications", Paweł Drygaś and Anna Król present new ways of constructing ordinal sums of fuzzy implication functions. The great number of field where these operators play an important role emphasizes the

necessity of having a large repertoire of fuzzy implication functions at one's disposal in order to pick out the operator with most convenient additional properties. An important method recently developed to construct new fuzzy implication functions from other given ones is the ordinal sum construction method. The authors of this paper propose two new possibilities of defining ordinal sums of fuzzy implications. The first one generalizes a previous method of the same authors by allowing summands on open, closed and half-open intervals. The second one constitutes a construction method that generates a fuzzy implication function without any additional assumption on the summands.

The third paper, entitled "Some New Solutions of the Distributivity Law $I(x, S(y, z)) = S(I(x, y), I(x, z))$ among R-implications and Triangular Conorms", which is written by Wanda Niemyska and Michał Baczyński, deals with the important topic of distributivity equations of fuzzy implication functions over different fuzzy logic connectives. Its importance in Fuzzy Control and Fuzzy Systems towards the reduction of the complexity of fuzzy "IF-THEN" rules has led to an extensive research in the last decades. In this paper, the distributivity law $I(x, S(y, z)) = S(I(x, y), I(x, z))$ where $I$ is a fuzzy implication function and $S$ a t-conorm is further investigated. Namely, the case when $I$ is an R-implication generated from a strict t-norm and $S$ is just a t-conorm or even a more general operator is analyzed. New solutions are presented in which $S$ is not an Archimedean t-conorm leading to new results which generalize some previous theorems presented by Baczyński and Jayaram in 2009.

We strongly believe that this special session will be a valuable contribution to the growing field of fuzzy implication functions and it will inspire many successors.