SS12: Probability with Fuzziness and Statistical Applications

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Traditionally, the theoretical framework for Statistics has been that of probability theory based on crisp events and crisp data. This session is about the interplay of probabilistic and fuzzy notions to extend the scope of probability theory, and how this can enhance statistical theory, methodology, and practice. In the session proposal, five broad categories were outlined.

Extensions of statistical methods to handle fuzzy data.

Fuzzy concepts for analyzing crisp (and also fuzzy) data.

Fuzzy random variables and other hybrid probabilistic-fuzzy uncertainty models.

Fuzziness in probability theory.

Interface topics (like copulas, statistical depth functions, functional data, large deviations).

The interface of Probability Theory and Fuzzy Set Theory is a classical topic dating back to Zadeh's probability of fuzzy events in the late 60s and the notions of probabilistic sets and fuzzy random variables in the 70s. Following up earlier special sessions in SMPS 2012, IFSA-EUSFLAT 2015, and SMPS 2016, this session aims at presenting the latest developments at the intersection of probability theory, fuzzy sets, and possibility theory, and their applications to statistics and data analysis. A serious, yet sadly unfruitful, effort was made to invite statisticians outside the fuzzy community who work on interface topics.

The paper *Construction of Fuzzy Multiple Deferred State Sampling Plan* by Robab Afshari and Bahram Sadeghpour Gildeh replaces the proportion of defective items p in acceptance sampling for quality control by a fuzzy quantity. Multiple deferred sampling is based on inspection of succesive item lots. If a lot has 'few' ('many') defective items, it is accepted (rejected); but if it has an intermediate number of defective items, the decision is deferred until we see whether the next lots are accepted or rejected. If a fuzzy p is used, under appropriate assumptions one obtains an approximate fuzzy binomial distribution for the acceptance probability.

The authors make the explicit calculations for the case that a lot is accepted if all sampled items are correct, rejected if 2 defective items are found, and deferred if 1 defective item is found. Based on a comparison of operator characteristic curves, the authors propose that the performance of the fuzzy multiple deferred sampling plan is better than that of fuzzy simple sampling.

In the paper *Knowledge Processing in Decisions Under Fuzziness and Uncertainty* by Giulianella Coletti and Barbara Vantaggi, the authors continue the systematic study of member-

ship functions and possibility measures in the context of coherent conditioning and thus, widely speaking, in the framework of De Finetti's ideas about probability.

They intend to formalize the interpretation of the membership function μ of a fuzzy set describing property $\mathcal{P}$ with $\mu(x)$ being the degree of belief that an object has property $\mathcal{P}$ if its value of the variable is x. The degree of belief can be expressed both possibilistically and probabilistically. This leads to the problem of identifying the coherent extensions of such belief assessments and the problem of which set-theoretical operations on fuzzy sets satisfy the requirement of respecting coherence. Multiplicity of extensions leads to considering upper and lower envelopes.

By reinterpreting concepts of fuzzy sets in a setting compatible with Bayesian reasoning, it is possible to incorporate fuzzy information in a Bayesian decision workflow. An example is given showing how to use fuzzy partitions in a model selection problem solved using Bayes factors.

The paper *Interval-Valued Risk Measure Models and Empirical Analysis* by Zihe Li, Jinping Zhang and Xiaoying Wang extends two risk measures in financial stochastics, the value at risk and the conditional value at risk, to the case when returns are modelled by random intervals instead of random variables. This research might be naturally extended in the future to fuzzy returns. Since the definitions of the risk measures require comparing returns to fixed thresholds, the extension needs a total preorder between intervals, which is given by a lexicographic comparison of the means and left end-points. In a fuzzy generalization, it would be interesting to choose an appropriate preorder between fuzzy intervals, among all the available possibilities in the literature.

The authors present an example with real data of four stocks from the Chinese stock exchanges, including bootstrap confidence intervals for the VaR and CVaR and the interval-valued VaR and CVaR as obtained from the daily lower and upper prices.